Write your name here

Surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | | | | | |

Candidate Number | | | | |

Further Mathematics

Advanced Subsidiary
Further Mathematics options
Paper 2H: Further Mechanics 1 and Decision Mathematics 1

Sample Assessment Material for first teaching September 2017

Time: 1 hour 40 minutes

Paper Reference

8FM0/2H

You must have:
Decision Mathematics 1 question insert for Section B
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- There are **two** sections in this question paper. Answer **all** the questions in Section A and **all** the questions in Section B.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 80.
- The questions for Section B (Decision Mathematics) can be found in the question insert.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

S58534A

1/1/1/1/1/1/

S 5 8 5 3 4 A 0 1 1 8

Pearson

SECTION A

Answer ALL questions. Write your answers in the spaces provided.

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\,\text{m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

1. A small ball of mass 0.1 kg is dropped from a point which is 2.4 m above a horizontal floor. The ball falls freely under gravity, strikes the floor and bounces to a height of 0.6 m above the floor. The ball is modelled as a particle.

(a) Show that the coefficient of restitution between the ball and the floor is 0.5

(6)

(b) Find the height reached by the ball above the floor after it bounces on the floor for the second time.

(3)

(c) By considering your answer to (b), describe the subsequent motion of the ball.

(1)

Question 1 continued

(Total for Question 1 is 10 marks)

2. A small stone of mass 0.5 kg is thrown vertically upwards from a point A with an initial speed of 25 m s^{-1}. The stone first comes to instantaneous rest at the point B which is 20 m vertically above the point A. As the stone moves it is subject to air resistance. The stone is modelled as a particle.

(a) Find the energy lost due to air resistance by the stone, as it moves from A to B.

(3)

The air resistance is modelled as a constant force of magnitude R newtons.

(b) Find the value of R.

(2)

(c) State how the model for air resistance could be refined to make it more realistic.

(1)

Question 2 continued

(Total for Question 2 is 6 marks)

3. [*In this question use* $g = 10\,m\,s^{-2}$]

A jogger of mass 60 kg runs along a straight horizontal road at a constant speed of $4\,m\,s^{-1}$. The total resistance to the motion of the jogger is modelled as a constant force of magnitude 30 N.

(a) Find the rate at which the jogger is working.

(3)

The jogger now comes to a hill which is inclined to the horizontal at an angle α, where $\sin\alpha = \frac{1}{15}$. Because of the hill, the jogger reduces her speed to $3\,m\,s^{-1}$ and maintains this constant speed as she runs up the hill. The total resistance to the motion of the jogger from non-gravitational forces continues to be modelled as a constant force of magnitude 30 N.

(b) Find the rate at which she has to work in order to run up the hill at $3\,m\,s^{-1}$.

(5)

Question 3 continued

(Total for Question 3 is 8 marks)

4. A particle P of mass $3m$ is moving in a straight line on a smooth horizontal table. A particle Q of mass m is moving in the opposite direction to P along the same straight line. The particles collide directly. Immediately before the collision the speed of P is u and the speed of Q is $2u$. The velocities of P and Q immediately after the collision, measured in the direction of motion of P before the collision, are v and w respectively. The coefficient of restitution between P and Q is e.

(a) Find an expression for v in terms of u and e.

(6)

Given that the direction of motion of P is changed by the collision,

(b) find the range of possible values of e.

(2)

(c) Show that $w = \frac{u}{4}(1 + 9e)$.

(2)

Following the collision with P, the particle Q then collides with and rebounds from a fixed vertical wall which is perpendicular to the direction of motion of Q. The coefficient of restitution between Q and the wall is f.

Given that $e = \frac{5}{9}$, and that P and Q collide again in the subsequent motion,

(d) find the range of possible values of f.

(6)

Question 4 continued

Question 4 continued

Question 4 continued

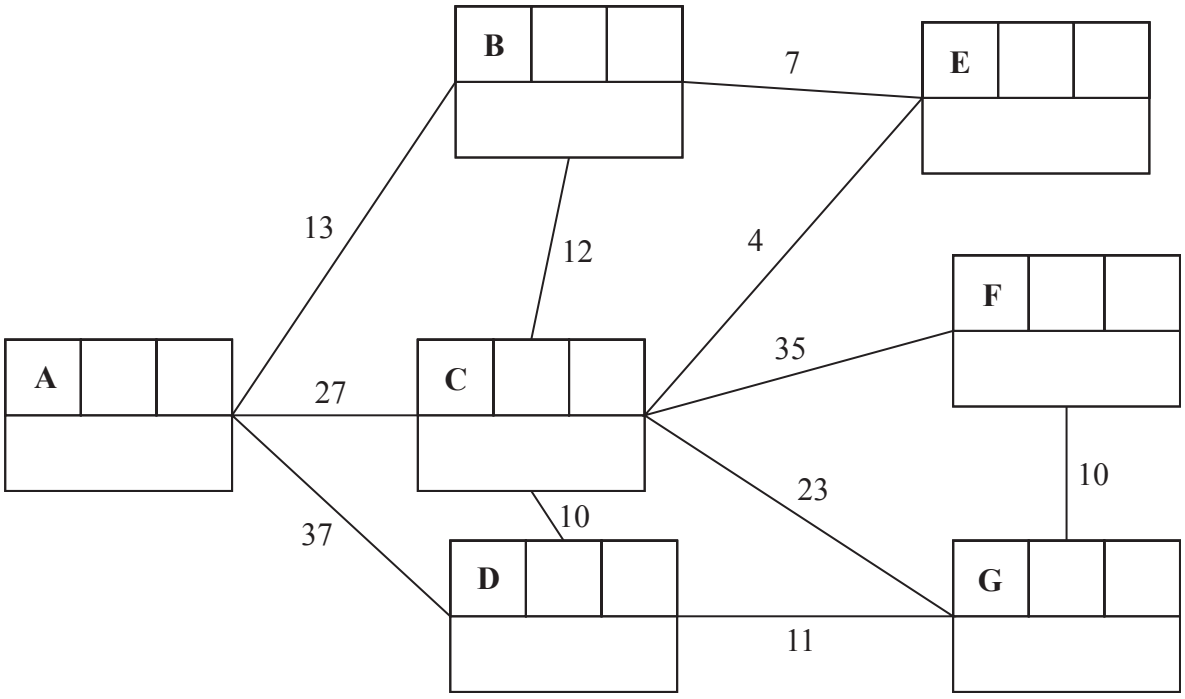
(Total for Question 4 is 16 marks)

TOTAL FOR SECTION A IS 40 MARKS

SECTION B

The questions for this section, Decision Mathematics 1, are provided in the Decision Mathematics 1 question insert.

5.



Key:

Vertex	Order of labelling	Final value
Working values		

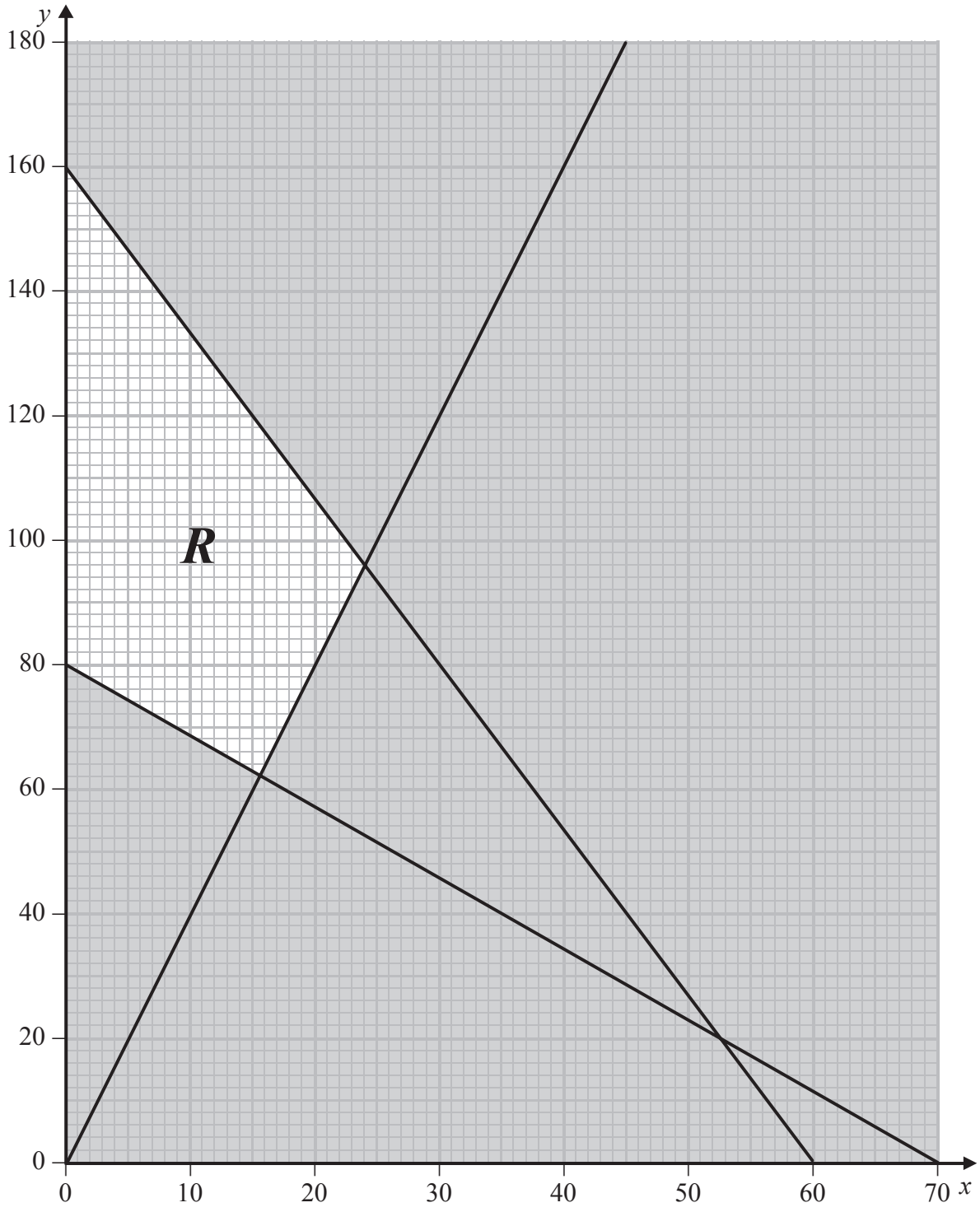
Shortest path: _____

Length of shortest path: _____

Question 5 continued

(Total for Question 5 is 12 marks)

6.



Question 6 continued

(Total for Question 6 is 7 marks)

7. (a) and (b)

(Total for Question 7 is 7 marks)

8.

(Total for Question 8 is 9 marks)

9.

(Total for Question 9 is 5 marks)

TOTAL FOR SECTION B IS 40 MARKS
TOTAL FOR PAPER IS 80 MARKS

Pearson Edexcel Level 3 GCE

Further Mathematics

Advanced Subsidiary
Further Mathematics options
Paper 2H: Section B Decision Mathematics 1

Sample Assessment Material for first teaching September 2017

Paper Reference
8FM0/2H

Decision Mathematics 1 question insert for Section B
Do not return this document with the question paper.

Turn over ▶

S58534A

1/1/1/1/1/1/

S 5 8 5 3 4 A 0 1 0 6

Pearson

SECTION B

Answer ALL questions. Write your answers in the answer book provided.

5.

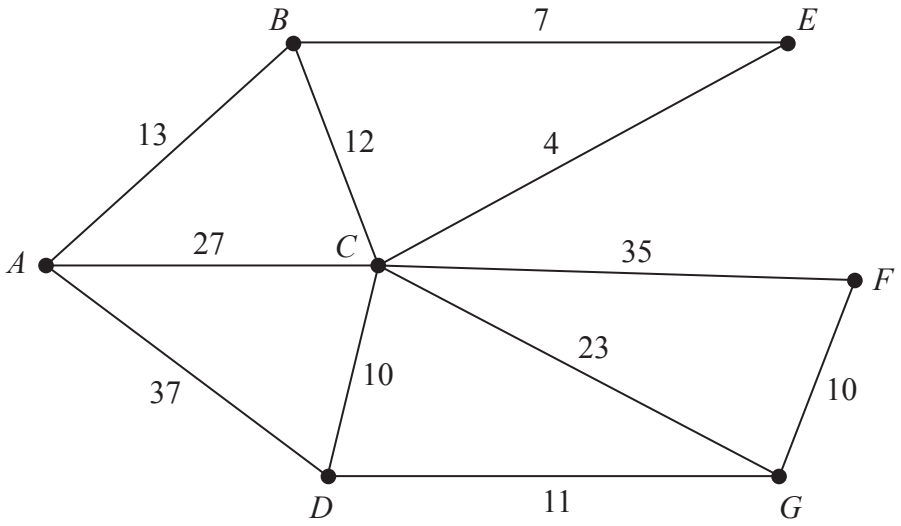


Figure 1

[*The total weight of the network is 189*]

Figure 1 represents a network of pipes in a building. The number on each arc is the length, in metres, of the corresponding pipe.

(a) Use Dijkstra's algorithm to find the shortest path from A to F. State the path and its length.

(5)

On a particular day, Gabriel needs to check each pipe. A route of minimum length, which traverses each pipe at least once and which starts and finishes at A, needs to be found.

(b) Use an appropriate algorithm to find the pipes that will need to be traversed twice. You must make your method and working clear.

(4)

(c) State the minimum length of Gabriel's route.

(1)

A new pipe, BG, is added to the network. A route of minimum length that traverses each pipe, including BG, needs to be found. The route must start and finish at A.

Gabriel works out that the addition of the new pipe increases the length of the route by twice the length of BG.

(d) Calculate the length of BG. You must show your working.

(2)

(Total for Question 5 is 12 marks)

6.

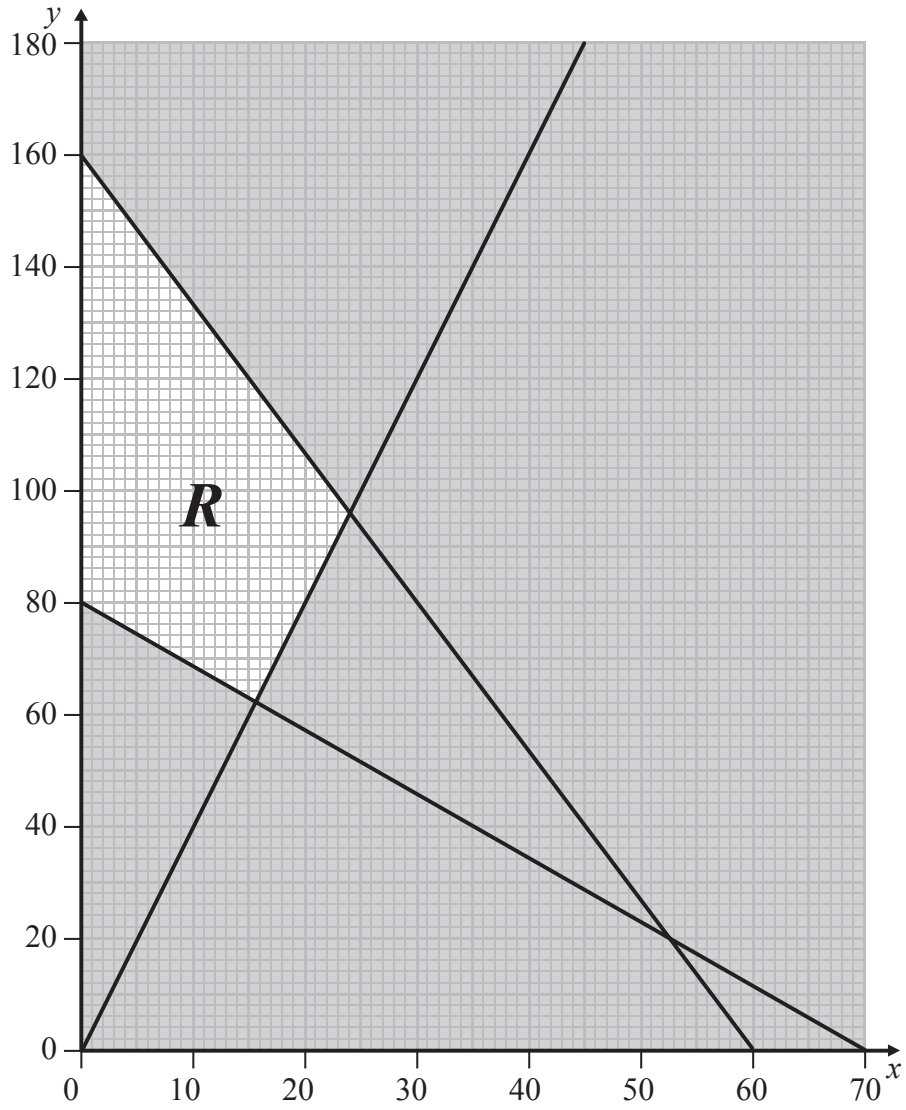


Figure 2

A teacher buys pens and pencils. The number of pens, x, and the number of pencils, y, that he buys can be represented by a linear programming problem as shown in Figure 2, which models the following constraints:

$$8x + 3y \leqslant 480$$

$$8x + 7y \geqslant 560$$

$$y \geqslant 4x$$

$$x, y \geqslant 0$$

The total cost, in pence, of buying the pens and pencils is given by

$$C = 12x + 15y$$

Determine the number of pens and the number of pencils which should be bought in order to minimise the total cost. You should make your method and working clear.

(Total for Question 6 is 7 marks)

7.

Activity	Time taken (days)	Immediately preceding activities
A	5	-
B	7	-
C	3	-
D	4	A, B
E	4	D
F	2	B
G	4	B
H	5	C, G
I	10	C, G

The table above shows the activities required for the completion of a building project. For each activity, the table shows the time taken in days to complete the activity and the immediately preceding activities. Each activity requires one worker. The project is to be completed in the shortest possible time.

(a) Draw the activity network described in the table, using activity on arc. Your activity network must contain the minimum number of dummies only.

(3)

(b) (i) Show that the project can be completed in 21 days, showing your working.

(ii) Identify the critical activities.

(4)

(Total for Question 7 is 7 marks)

8. (a) Explain why it is not possible to draw a graph with exactly 5 nodes with orders 1, 3, 4, 4 and 5

(1)

A connected graph has exactly 5 nodes and contains 18 arcs. The orders of the 5 nodes are $2^{2x} - 1$, 2^x, $x + 1$, $2^{x+1} - 3$ and $11 - x$.

(b) (i) Calculate x.

(ii) State whether the graph is Eulerian, semi-Eulerian or neither. You must justify your answer.

(6)

(c) Draw a graph which satisfies all of the following conditions:

- The graph has exactly 5 nodes.
- The nodes have orders 2, 2, 4, 4 and 4
- The graph is not Eulerian.

(2)

(Total for Question 8 is 9 marks)

9. Jonathan makes two types of information pack for an event, *Standard* and *Value*.

Each *Standard* pack contains 25 posters and 500 flyers.

Each *Value* pack contains 15 posters and 800 flyers.

He must use at least 150 000 flyers.

Between 35% and 65% of the packs must be *Standard* packs.

Posters cost 20p each and flyers cost 4p each.

Jonathan wishes to minimise his costs.

Let x and y represent the number of *Standard* packs and *Value* packs produced respectively.

Formulate this as a linear programming problem, stating the objective and listing the constraints as simplified inequalities with integer coefficients.

You should not attempt to solve the problem.

(Total for Question 9 is 5 marks)

TOTAL FOR SECTION B IS 40 MARKS

Paper 2 Option H

Further Mechanics 1 Mark Scheme (Section A)

Question	Scheme	Marks	AOs
1(a)	Using the model and $v^2 = u^2 + 2as$ to find v	M1	3.4
	$v^2 = 2as = 2g \times 2.4 = 4.8g \Rightarrow v = \sqrt{(4.8g)}$	A1	1.1b
	Using the model and $v^2 = u^2 + 2as$ to find u	M1	3.4
	$0^2 = u^2 - 2g \times 0.6 \Rightarrow u = \sqrt{(1.2g)}$	A1	1.1b
	Using the correct strategy to solve the problem by finding the sep. speed and app. speed and applying NLR	M1	3.1b
	$e = \sqrt{(1.2g)} / \sqrt{(4.8g)} = 0.5$ *	A1*	1.1b
		(6)	
(b)	Using the model and e = sep. speed / app. speed, $v = 0.5\sqrt{(1.2g)}$	M1	3.4
	Using the model and $v^2 = u^2 + 2as$	M1	3.4
	$0^2 = 0.25\,(1.2g) - 2gh \Rightarrow h = 0.15$ (m)	A1	1.1b
		(3)	
(c)	Ball continues to bounce with the height of each bounce being a quarter of the previous one	B1	2.2b
		(1)	
			(10 marks)

Notes:

(a)
M1: For a complete method to find v
A1: For a correct value (may be numerical)
M1: For a complete method to find u
A1: For a correct value (may be numerical)
M1: For finding both v and u and use of Newton's Law of Restitution
A1*: For the given answer

(b)
M1: For use of Newton's Law of Restitution to find rebound speed
M1: For a complete method to find h
A1: For 0.15 (m) oe

(c)
B1: For a clear description including reference to a quarter

Question	Scheme	Marks	AOs
2(a)	Energy Loss = KE Loss – PE Gain	M1	3.3
	$= \frac{1}{2} \times 0.5 \times 25^2 - 0.5\,g \times 20$	A1	1.1b
	$= 58.25 = 58$ (J) or 58.3 (J)	A1	1.1b
		(3)	
(b)	Using work-energy principle, $20\,R = 58.25$	M1	3.3
	$R = 2.9125 = 2.9$ or 2.91	A1ft	1.1b
		(2)	
(c)	Make resistance variable (dependent on speed)	B1	3.5c
		(1)	
			(6 marks)

Notes:

(a)
M1: For a difference in KE and PE
A1: For a correct expression
A1: For either 58 (2sf) or 58.3(3sf)

(b)
M1: For use of work-energy principle
A1ft: For either 2.9 (2sf) or 2.91 (3sf) follow through on their answer to (a)

(c)
B1: For variable resistance oe

Question	Scheme	Marks	AOs
3(a)	Force = Resistance (since no acceleration) = 30	B1	3.1b
	Power = Force × Speed = 30 × 4	M1	1.1b
	= 120 W	A1 **ft**	1.1b
		(3)	
(b)	Resolving parallel to the slope	M1	3.1b
	$F - 60g\sin\alpha - 30 = 0$	A1	1.1b
	$F = 70$	A1	1.1b
	Power = Force × Speed = 70 × 3	M1	1.1b
	= 210 W	A1 **ft**	1.1b
		(5)	
			(8 marks)

Notes:

(a)
B1: For force = 30 seen
M1: For use of $P = Fv$
A1ft: For 120 (W), follow through on their '30'

(b)
M1: For resolving parallel to the slope with correct no. of terms and 60*g* resolved
A1: For a correct equation
A1: For $F = 70$
M1: For use of $P = Fv$
A1ft: For 210 (W), follow through on their '70'

Question	Scheme	Marks	AOs
4(a)	Use of conservation of momentum	M1	3.1a
	$3mu - 2mu = 3mv + mw$	A1	1.1b
	Use of NLR	M1	3.1a
	$3ue = -v + w$	A1	1.1b
	Using a correct strategy to solve the problem by setting up two equations (need both) in u and v and solving for v	M1	3.1b
	$v=\frac{u}{4}(1 - 3e)$	A1	1.1b
		(6)	
(b)	$\frac{u}{4}(1 - 3e) < 0$	M1	3.1b
	$\frac{1}{3} < e \leq 1$	A1	1.1b
		(2)	
(c)	Solving for w	M1	2.1
	$w=\frac{u}{4}(1 + 9e)$ *	A1 *	1.1b
		(2)	
(d)	Substitute $e=\frac{5}{9}$	M1	1.1b
	$v = -\frac{u}{6}, w=\frac{3u}{2}$	A1	1.1b
	Use NLR for impact with wall, $x = fw$	M1	1.1b
	Further collision if $x > -v$	M1	3.4
	$f\frac{3u}{2} > \frac{u}{6}$	A1	1.1b
	$1 \geq f > \frac{1}{9}$	A1	1.1b
		(6)	
			(16 marks)

Notes:

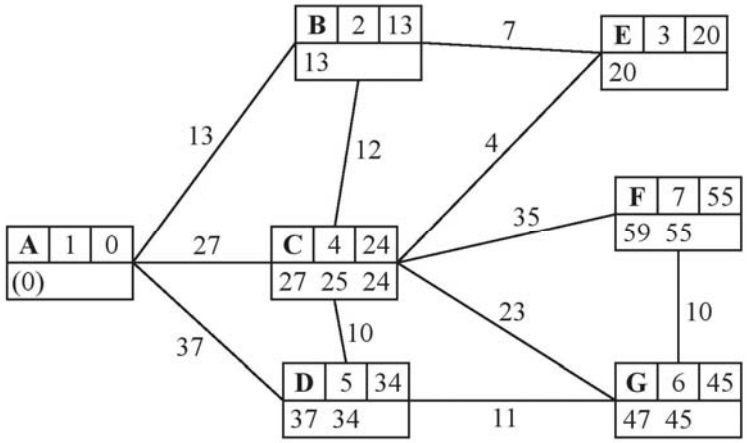
(a)
M1: For use of CLM, with correct no. of terms, condone sign errors
A1: For a correct equation
M1: For use of Newton's Law of Restitution, with e on the correct side
A1: For a correct equation
M1: For setting up *two* equations and solving their equations for v
A1: For a correct expression for v

(b)
M1: For use of an appropriate inequality
A1: For a complete range of values of e

(c)
M1: For solving their equations for w
A1: For the given answer

Question 4 notes continued:
(d)
M1: For substituting $e = \frac{5}{9}$ into their v and w
A1: For correct expressions for v and w
M1: For use of Newton's Law of Restitution, with e on the correct side
M1: For use of appropriate inequality
A1: For a correct inequality
A1: For a correct range

Decision Mathematics 1 Mark Scheme (Section B)

Question	Scheme	Marks	AOs
5(a)		M1 A1 A1	1.1b 1.1b 1.1b
	Path: ABECDGF	A1	1.1b
	Length: 55 (metres)	A1ft	1.1b
		(5)	
(b)	AB + DG = 13 + 11 = 24 ←	M1	1.1b
	A(BEC)D + B(ECD)G = 34 + 32 = 66	A1	1.1b
	A(BECD)G + B(EC)D = 45 + 21 = 66	A1	1.1b
	Repeat arcs: AB, DG	A1ft	2.2a
		(4)	
(c)	Length = 189 + 24 = 213 (metres)	B1ft	1.1b
		(1)	
(d)	$189 + x + 34 = 213 + 2x$	M1	3.1b
	$x = 10$ so BG is 10 m	A1	1.1b
		(2)	
			(12 marks)

Notes:

(a)
M1: For a larger number replaced by a smaller one in the working values boxes at C, D, F or G
A1: For all values correct (and in correct order) at A, B, C and D
A1: For all values correct (and in correct order) at E, F & G
A1: For the correct path
A1ft: For 55 or ft their final value at F

(b)
M1: For 3 correct pairings of the four odd nodes (A,B, D & G)
A1: At least two pairings and totals correct
A2: All three pairings and totals correct
A3ft: Selecting their shortest pairing, and stating that these arcs should be repeated

Question 5 notes continued:
(c) **B1ft:** For 213 or 189 + their shortest repeat **M1:** For translating the information in the question in to an equation involving x, $2x$ and 34 **A1:** For a correct equation leading to BG = 10 (m)

Question	Scheme	Marks	AOs
6	Objective line drawn or at least two vertices tested	M1	3.1a
	For solving y = 4x and 8x + 7y = 560 to find the exact co-ordinate of the optimal point, must reach either x = or y =	M1	1.1a
	$x = 15\frac{5}{9}$ and $y = 62\frac{2}{9}$	A1	1.1b
	Finding at least two points with integer co-ordinates from (15 ± 1, 63 ± 2)	M1	1.1b
	Testing at least two points with integer co-ordinates	M1	1.1b
	x = 15 and y = 63	A1	2.2a
	So the teacher should buy 15 pens and 63 pencils	A1ft	3.2a
			(7 marks)

Notes:

M1: Selecting an appropriate mathematical process to solve the problem – either drawing an objective line with the correct gradient (or reciprocal gradient), or testing at least two vertices in C

M1: Solving simultaneous equations

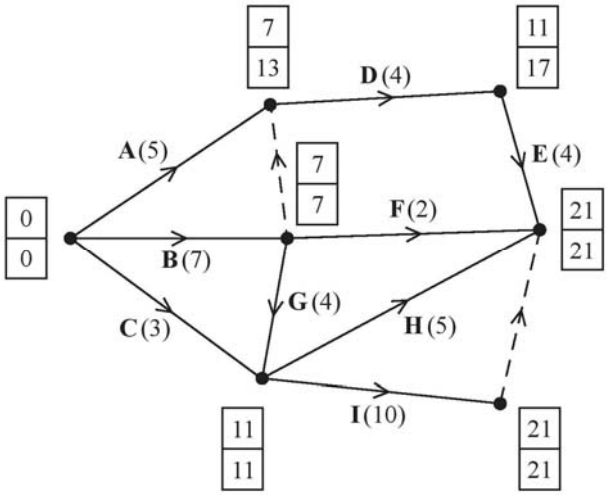
A1: cao

M1: Recognition that outcome from this model is non-integer and integer solutions are required – testing two points with integer co-ordinates in at least one of $y \geq 4x$ and $8x + 7y \geq 560$

M1: Testing at least two integer solutions in $y \geq 4x$ or $8x + 7y \geq 560$ and C

A1: cao – deducing from tests which integer solution is both valid and optimal

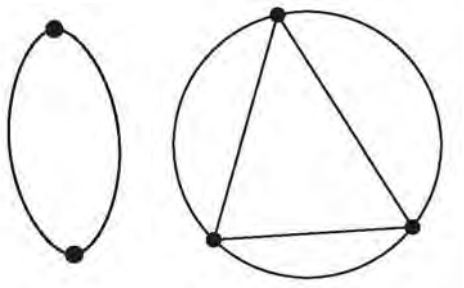
A1ft: Interpreting solution in the context of the question – gives their integer values for x and y in the context of pens and pencils

Question	Scheme	Marks	AOs
7(a)(b)		M1 A1 A1	1.1b 1.1b 1.1b
		(3)	
	The number(s) at the end of activity E indicate this project can be completed in 21 days	M1 A1 A1ft	2.1 1.1b 2.2a
	Critical activities: B, G, I	A1	1.1b
		(4)	
			(7 marks)

Notes:

M1: At least 5 activities and one dummy, one start
A1: A,B,C,D,F,G and first dummy correct
A1: E,H,I correct, second dummy correct and one finish

M1: All boxes completed, number generally increasing L to R (condone one "rogue")
A1: All values cao
A1: Deduction that result in diagram indicates that project can be completed in 21 days (all boxes completed, numbers generally increasing in the direction of the arrows for the top boxes and generally decreasing in the opposite direction of the arrow for the bottom boxes)
A1: Critical activities correct

Question	Scheme	Marks	AOs
8(a)	e.g. a graph cannot contain an odd number of odd nodes e.g. number of arcs $= \frac{1+3+4+4+5}{2} = 8.5 \notin \mathbb{Z}$	B1	2.4
		(1)	
(b)(i)	$\left(2^{2x}-1\right)+\left(2^{x}\right)+(x+1)+\left(2^{x+1}-3\right)+(11-x)=2(18)$	M1	1.1b
	$2^{2x}+3\left(2^{x}\right)-28=0 \Rightarrow x=...$	M1	1.1b
	$\left(2^{x}+7\right)\left(2^{x}-4\right)=0 \Rightarrow x=2$	A1	1.1b
		(3)	
(b)(ii)	The order of the nodes are 9, 15, 3, 4, 5	M1	2.1
	Therefore the graph is neither Eulerian nor semi-Eulerian as there are more than two odd nodes	A1	2.4
		A1	2.2a
		(3)	
(c)		M1 A1	2.5 2.2a
		(2)	
			(9 marks)

Notes:

(a)
B1: Explanation referring to need for an even number of odd nodes oe

(b)
M1: Forming an equation involving the orders of the 5 odd nodes and 2(18)
M1: Simplifies to a quadratic in 2^x and attempts to solve
A1: 2 cao
M1: Construct an argument involving the order of the 5 nodes
A1: Explanation considering the number of odd nodes
A1: Deduction that therefore it is neither Eulerian nor semi-Eulerian

(c)
M1: Interprets mathematical language to construct a disconnected graph
A1: Deduce a correct graph

Question	Scheme	Marks	AOs
9	Minimise $(C=)\ 25x+35y$	B1	3.3
	Subject to: $(500x+800y\geqslant 150\ 000\Rightarrow)\ 5x+8y\geqslant 1500$	B1	3.3
	$\frac{7}{20}(x+y)\leqslant x\leqslant\frac{13}{20}(x+y)$	M1 M1	3.3 3.3
	Which simplifies to $7y\leqslant 13x$ **and** $13y\geqslant 7x$ $x, y\geqslant 0$	A1	1.1b
			(5 marks)

Notes:

B1: A correct objective function + minimise
B1: Translate information in to a correct inequality
M1: For translating the information given into the LHS inequality
M1: For translating the information given in to the RHS inequality
A1: Simplifying to the correct inequalities